Fundamentals of interactions of particles in matter

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Research for a Life without Cancer

Bibliography

- Techniques for Nuclear and Particle Physics: A How-to Approach, William R. Leo, Springer-Verlag, 2nd Ed., Ch. 2, pp. 17-53
- Principles of Radiation Interaction in Matter and Detection, Claude Leroy and Pier-Giorgio Rancoita, World Scientific, Ch. 1-2, pp. 19-135
- Physics of Proton Interactions in Matter, Bernhard Gottschalk, in: Proton Therapy Physics, Harald Paganetti, Ch. 2, pp. 20-59

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- Radiation Detection and Measurement, Glenn F. Knoll, Wiley, 4th Ed., Ch. 2, pp. 29-42
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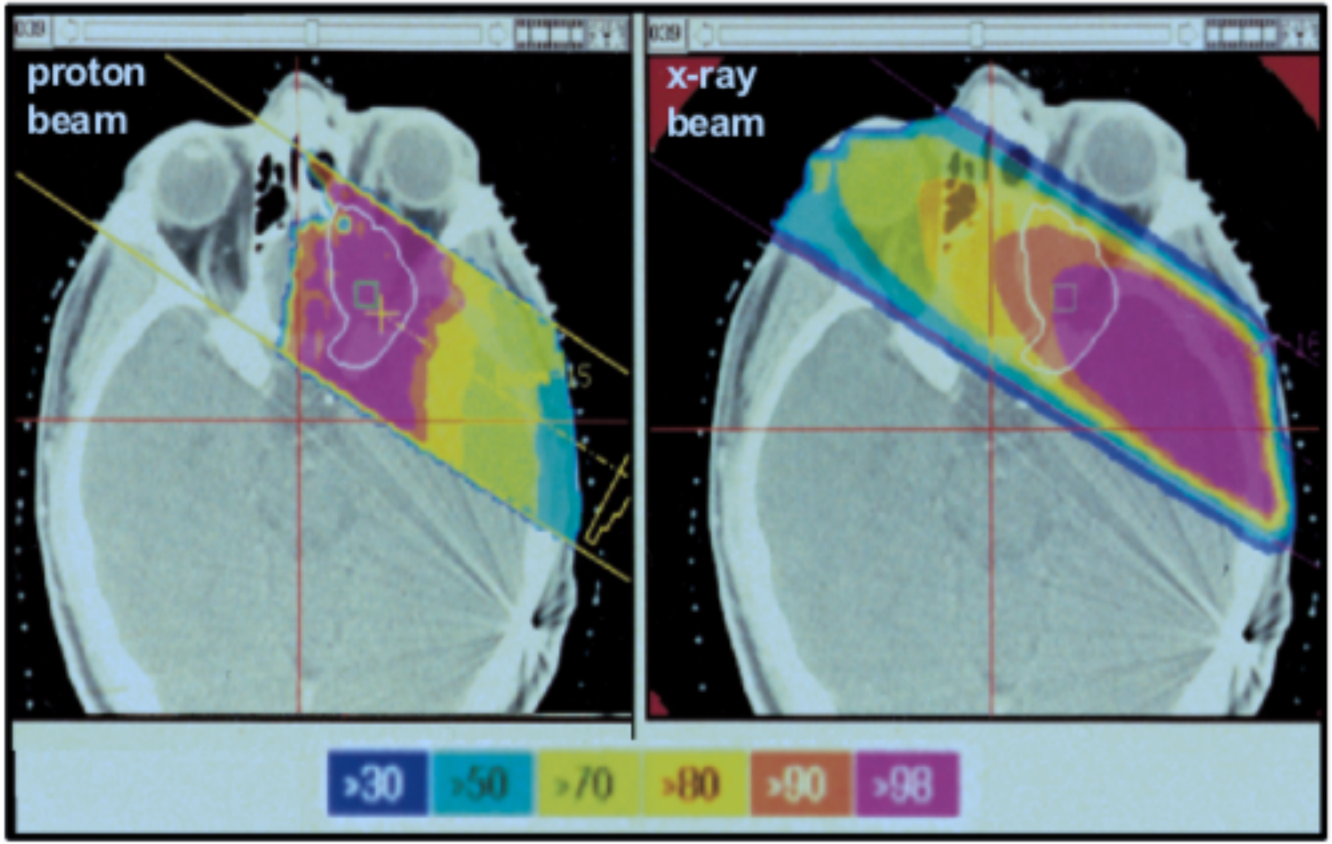
Goal

Two physics problems in particle radiotherapy:

- **Designing beam lines**
- **Predicting the dose distribution in the patient**

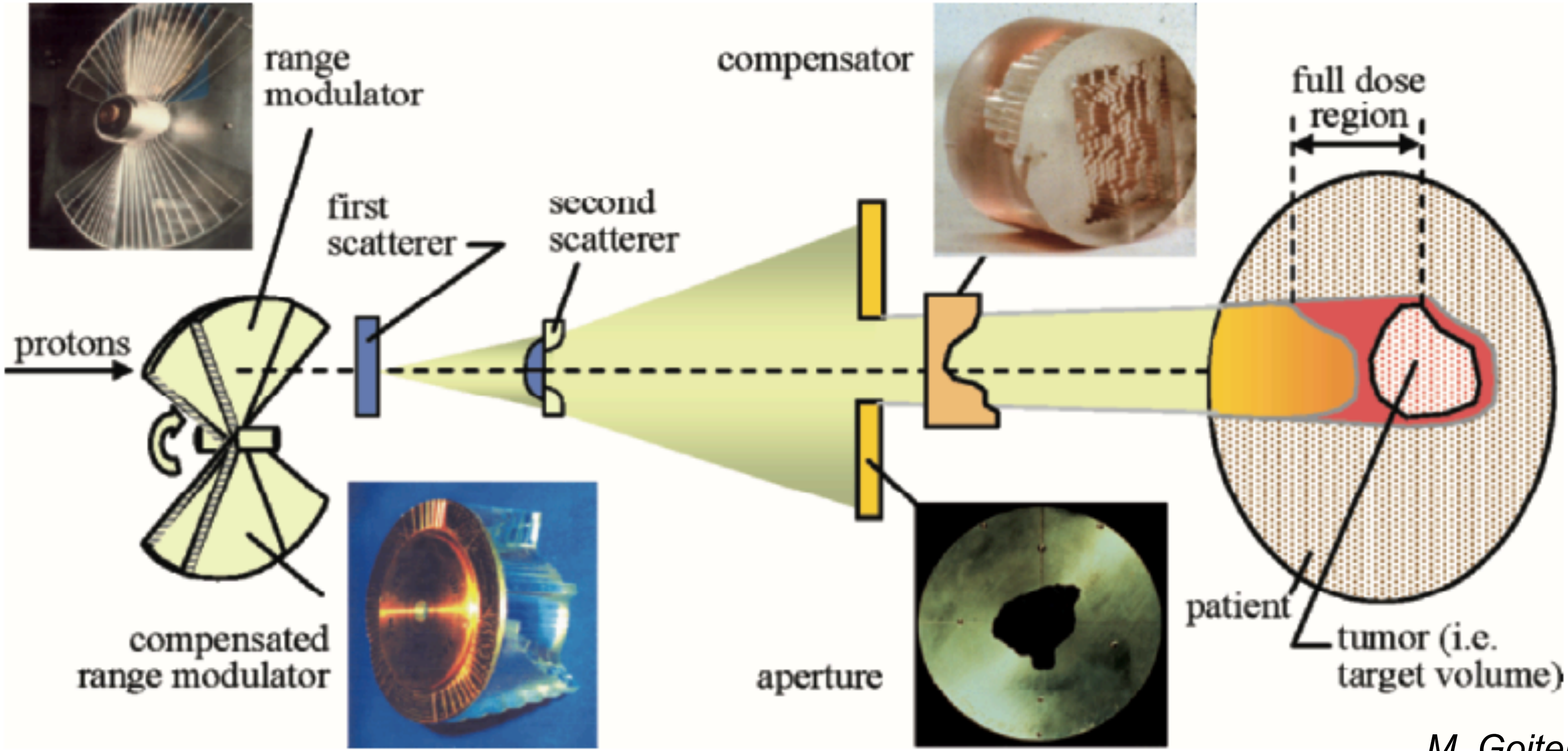
We need to understand the interactions of particles with matter



Proton and a photon posterior-oblique beam

M. Goitein 2008

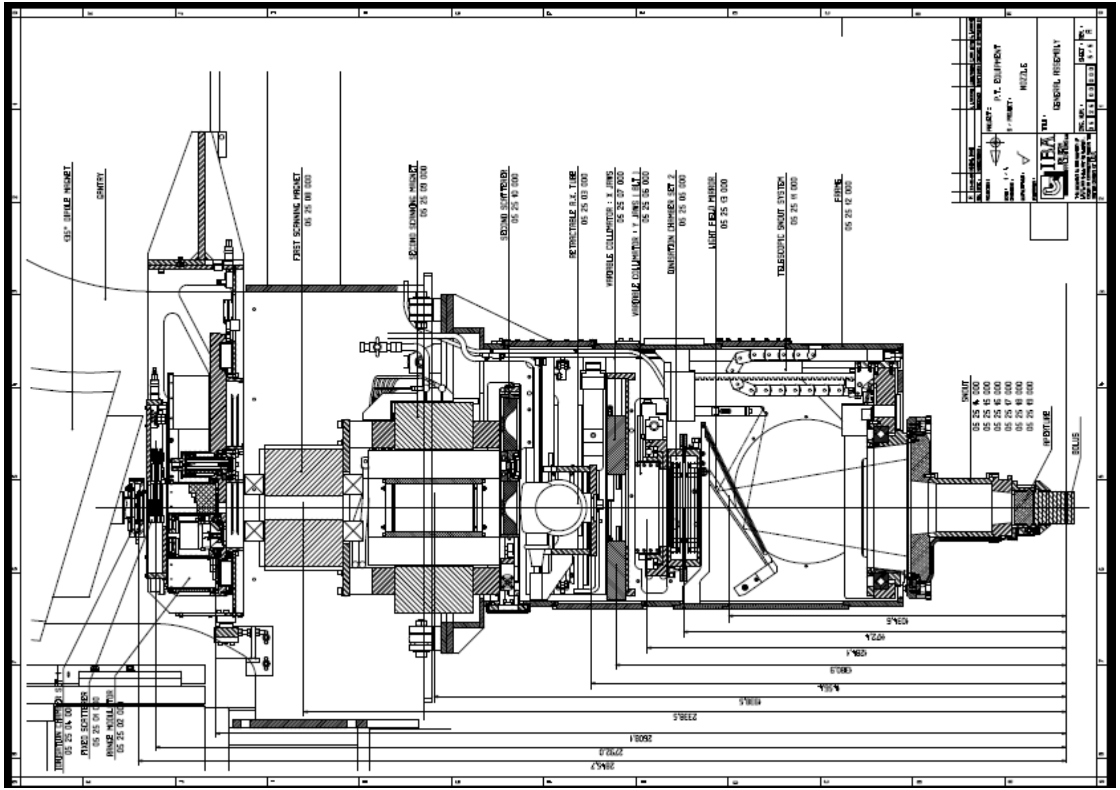
Accelerator and beam delivery (passive scattering)



M. Goitein 2008

Accelerator and beam delivery (passive scattering)

IBA Nozzle (Burr Center Gantry)



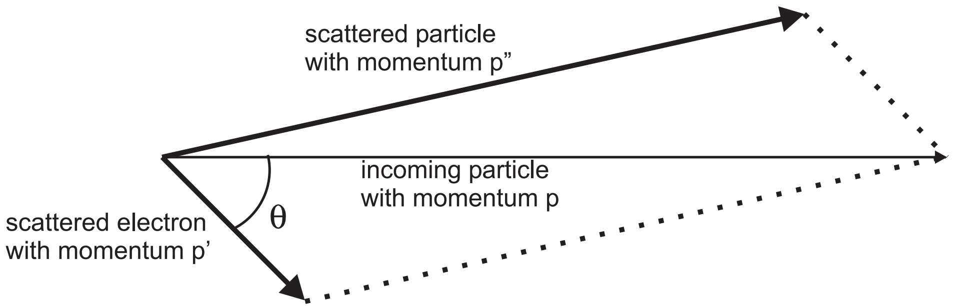
B. Gottschalk 2007

Outline

- Preliminary notions and definitions
- Energy loss of heavy charged particles by atomic collisions
- The Bethe-Bloch formula (stopping power)
- Fluence, stopping power and dose
- Range

Preliminary notions and definitions

Two-body scattering and maximum energy transfer



Relation kinetic energy momentum (scattered particle)

$$E_k + m_e c^2 = \sqrt{p'^2 c^2 + m_e^2 c^4}$$

Energy and momentum are conserved:

$$\sqrt{p^2 c^2 + m^2 c^4} + m_e c^2 = \sqrt{p''^2 c^2 + m^2 c^4} + E_k + m_e c^2$$

$$\vec{p}'' = \vec{p} - \vec{p}' \qquad \Longrightarrow \qquad p''^2 = p^2 + p'^2 - 2p\,p' \cos\theta$$

C. Leroy & P.G. Rancoita, (2009) Radiation Interaction in Matter and Detection, World Scientific

Preliminary notions and definitions

Two-body scattering and maximum energy transfer

Kinetic energy of the scattered target particle:

$$E_k = \frac{2m_e c^4 p^2 \cos^2\theta}{\left(m_e c^2 + \sqrt{p^2c^2 + m^2c^4}\right)^2 - p^2c^2\cos^2\theta}$$

Maximum energy transfer W_m is for $\theta = 0$ (head-on collision):

$$W_m = \frac{p^2c^2}{\frac{1}{2}m_e c^2 + \frac{1}{2}\left(m^2/m_e\right)c^2 + \sqrt{p^2c^2 + m^2c^4}}$$

dkfz.

Preliminary notions and definitions

Two-body scattering and maximum energy transfer

Since: $E_i = m\gamma c^2 = \sqrt{p^2c^2 + m^2c^4}$

$$W_m = 2m_ec^2\beta^2\gamma^2\left[1 + \left(\frac{m_e}{m}\right)^2 + 2\gamma\frac{m_e}{m}\right]^{-1}$$

For particles $m \gg m_e$

$$W_m \approx 2m_ec^2\frac{\beta^2}{1-\beta^2} = 2m_ec^2\beta^2\gamma^2$$

(valid for $2\gamma m_e \ll M$)

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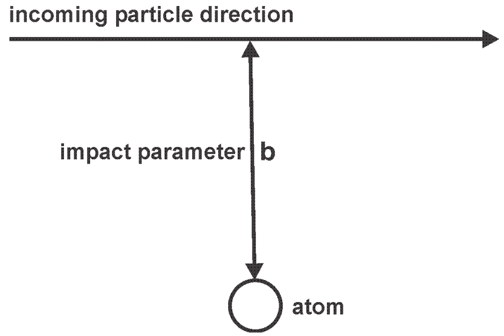
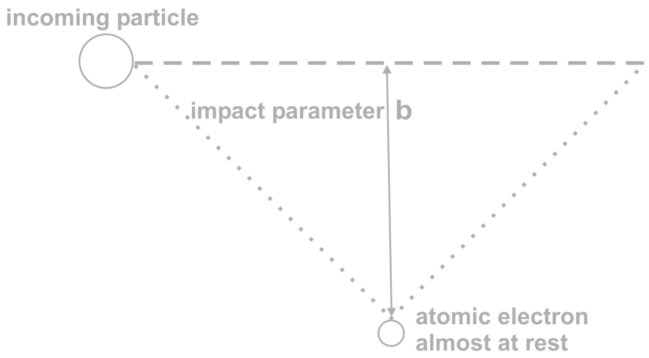
Energy loss of heavy charged particles by atomic collisions

Bohr’s calculation – the classical case



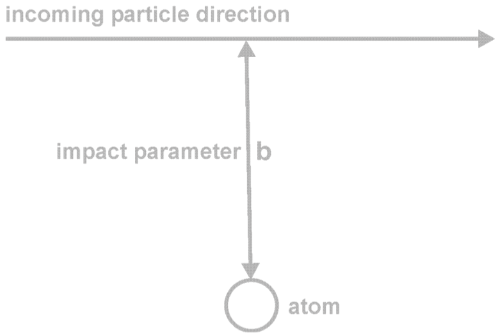


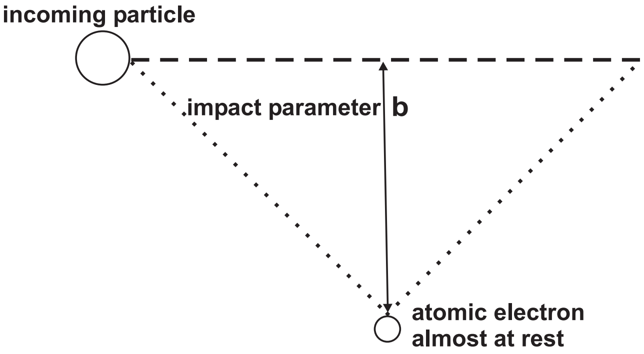
The impact parameter *b* is the minimum distance between the incoming particle and the target by which it is scattered

C. Leroy & P.G. Rancoita, (2009) Radiation Interaction in Matter and Detection, World Scientific

Energy loss of heavy charged particles by atomic collisions

Bohr's calculation – the classical case





The incoming fast particle of charge ze is not deflected by an atomic electron almost at rest for small energy transfer

C. Leroy & P.G. Rancoita, (2009) Radiation Interaction in Matter and Detection, World Scientific

Energy loss of heavy charged particles by atomic collisions

Bohr's calculation – the classical case

The transferred momentum $I_{\perp}$ will be almost along the direction perpendicular to the particle trajectory

$$F_{\perp} \approx ze^2/b^2$$

$$I_{\perp} = \int F_{\perp}\, dt \sim \frac{ze^2}{bv}$$

More momentum is transferred to the electron, the longer the particle stays in its vicinity

where the interaction time is $\approx \frac{b}{v}$

inversely proportional to the incoming particle velocity and directly proportional to b

Energy loss of heavy charged particles by atomic collisions

Bohr's calculation – the classical case

Relationship between the impact parameter *b* and the transferred energy *W*

Kinetic energy :

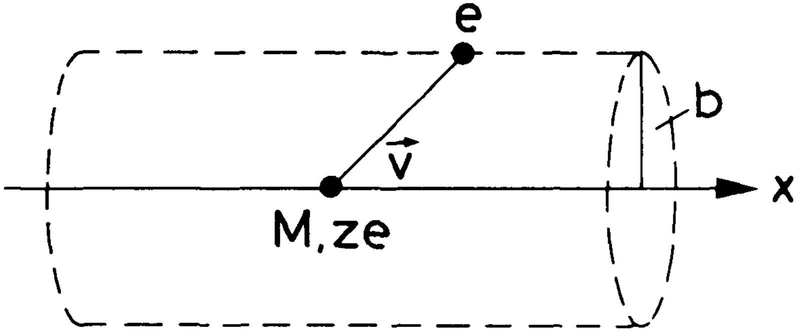
$$W \sim \frac{I_{\perp}^{2}}{2m} = \frac{2z^{2}e^{4}}{mb^{2}v^{2}}$$

Distant collisions are soft ones, while close collisions allow large transfers of kinetic energy (energy transferred to recoiling nuclei can be usually neglected with respect to the one from recoiling electrons – ratio 10^{-4})

Energy loss of heavy charged particles by atomic collisions

Bohr's calculation – the classical case

W.R. Leo, (1994) Techniques for Nuclear and Particle Physics Experiments, Springer-Verlag



Let *n* be the density of electrons, then the energy lost to all the electrons located at a distance between *b* and *b* + *db* in a thickness *dx is:*

$$-dE(b) = \Delta E(b)\, n\, dV = \frac{4\pi n z^2 e^4}{mv^2}\frac{db}{b}dx$$

where $n\, dV = n\,(2\pi b) db\, dx$ are the number of electrons encountered by the particle along a path dx at impact parameter between b and $b + db$

Energy loss of heavy charged particles by atomic collisions

Bohr's calculation – the classical case

The overall energy loss by collisions calculated by integrating over the range of the impact parameter is:

$$-\frac{dE}{dx} = \int_{b_{min}}^{b_{max}} \frac{4\pi n z^2 e^4}{mv^2} \frac{db}{b} = \frac{4\pi n z^2 e^4}{mv^2} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

Energy loss of heavy charged particles by atomic collisions

Bohr's calculation – the classical case

Limits:

$$b_{max} \simeq \frac{v\gamma}{\bar{\upsilon}}$$

where $\bar{\upsilon}$ is the characteristic mean frequency of excitation of electrons;

and $\tau \simeq \left(\frac{1}{\bar{\upsilon}}\right)$ is the collision time that cannot exceed the typical time period associated with bound electrons. This is the principle of adiabatic invariance - the perturbation must not be adiabatic, otherwise no energy is transferred.

Energy loss of heavy charged particles by atomic collisions

Bohr's calculation – the classical case

Limits:

$$b_{max} \simeq \frac{v\gamma h}{I}$$

if we introduce the mean excitation energy $I = h\bar{v}$

Energy loss of heavy charged particles by atomic collisions

Bohr's calculation – the classical case

Limits:

$$b_{min} \simeq \frac{h}{2P_{ecm}} \ ,$$

where P_{ecm} is the electron momentum in the CMS

In the classical approach, the wave characteristics of particles are neglected and this is valid as long as the impact parameter *b* is larger than the de Broglie wavelength of the electron in the CMS of the interaction

Energy loss of heavy charged particles by atomic collisions

Bohr's calculation – the classical case

Limits:

$$\left| P_{ecm} \right| \simeq m\gamma v = m\gamma\beta c \Longrightarrow b_{min} \simeq \frac{h}{2m\gamma\beta c}.$$

The electron mass is much smaller than the mass of the incoming particle and the CMS is approximately associated with the incoming particle. The electron velocity in the CMS is opposite and almost equal to the $v_{particle}$.

Energy loss of heavy charged particles by atomic collisions

Bohr's calculation – the classical case

Substituting:

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{mv^2} \ln\left(\frac{b_{max}}{b_{min}}\right) = \frac{4\pi n z^2 e^4}{mv^2} \ln\left[\left(\frac{v\gamma h}{I}\right)\left(\frac{2m\gamma\beta c}{h}\right)\right]$$

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{mv^2} \ln\left(\frac{2m\gamma^2 v^2}{I}\right)^2$$

Energy loss of heavy charged particles by atomic collisions

Bohr’s calculation – the classical case

Using the value of the maximum energy transfer W_m:

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{m v^2} \ln\left(\frac{2 m v^2 W_m}{I^2(1-\beta^2)}\right)$$

This is essentially **Bohr's classical formula**. It gives a reasonable description of the energy loss for heavy particles such as the **α-particle** or **heavier nuclei**. However, for lighter particles, e.g. the proton, the formula breaks down because of quantum effects.

Outline

- Preliminary notions and definitions
- Energy loss of heavy charged particles by atomic collisions
- The Bethe-Bloch formula (stopping power)
- Fluence, stopping power and dose
- Range

The Bethe-Bloch formula (Stopping Power)

Bethe relativistic formula (energy-loss formula)

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{mv^2}\left[\ln\left(\frac{2mv^2}{I^2(1-\beta^2)}\right) - \beta^2\right]$$

$$W_m \approx 2m_e c^2 \frac{\beta^2}{1-\beta^2} = 2m_e c^2 \beta^2 \gamma^2$$

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{mv^2} \ln\left(\frac{2mv^2 W_m}{I^2(1-\beta^2)}\right) - 2\beta^2$$

The Bethe-Bloch formula (Stopping Power)

Energy-loss formula (rewritten) and with corrections

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{mv^2}\left[\ln\left(\frac{2mv^2\gamma^2}{I}\right) - \beta^2 - \frac{\delta}{2} - \frac{U}{2}\right]$$

$\frac{\delta}{2}$ Density-effect correction

$\frac{U}{2}$ Shell effect correction

The logarithmic term increases quadratically with $\beta\gamma = (v\gamma)/c$

The Bethe-Bloch formula (Stopping Power)

Features:

- Energy loss depends on ln(b_{max}/b_{min}); $1/b_{min}$ and b_{max} increase with $\beta\gamma$
- $1/b_{min}$ -> enhancement of the maximum transferable kinetic energy
- b_{max} -> dilation of the maximum impact parameter
- With increasing particle energy, δ-rays are emitted along the trajectory
- Cylindrical region surrounding the particle path is enlarged
- Emission of δ-rays is responsible for the difference between the energy lost by the particle in the medium and the actually energy deposited.
- With increasing particle energy, the deposited energy approaches an almost constant value (*Fermi Plateau*) - depends on absorber size and density

The Bethe-Bloch formula (Stopping Power)

Energy-loss formula (rewritten)

classical electron radius

$$\frac{4\pi n z^2 e^4}{m v^2} = \frac{2\pi N m c^2 \boxed{r_e^2} \rho Z}{A\beta^2} = 0.1535\,\frac{\rho z^2 Z}{A\beta^2}$$

$$-\frac{dE}{dx} = 0.1535\,\frac{\rho z^2 Z}{A\beta^2}\left[\ln\left(\frac{2mv^2 W_m}{I^2(1-\beta^2)}\right) - \beta^2 - \frac{\delta}{2} - \frac{U}{2}\right]$$

Mean excitation energy:

$$\frac{I}{Z} = 12 + \frac{7}{Z}, Z < 13 \qquad \frac{I}{Z} = 9.76 + 58.8\,Z^{-1.19}, Z \geq 13$$

The Bethe-Bloch formula (Stopping Power)

Mass stopping power (radiotherapy energy 3-300 MeV)

$$\frac{S_{el}}{\rho} = \frac{1}{\rho}\frac{dE}{dx} = 0.3072\,\frac{Z}{A\beta^2}\left[\ln\left(\frac{W_m}{I}\right) - \beta^2\right] \quad \frac{\mathrm{MeV}}{\mathrm{g/cm^2}}$$

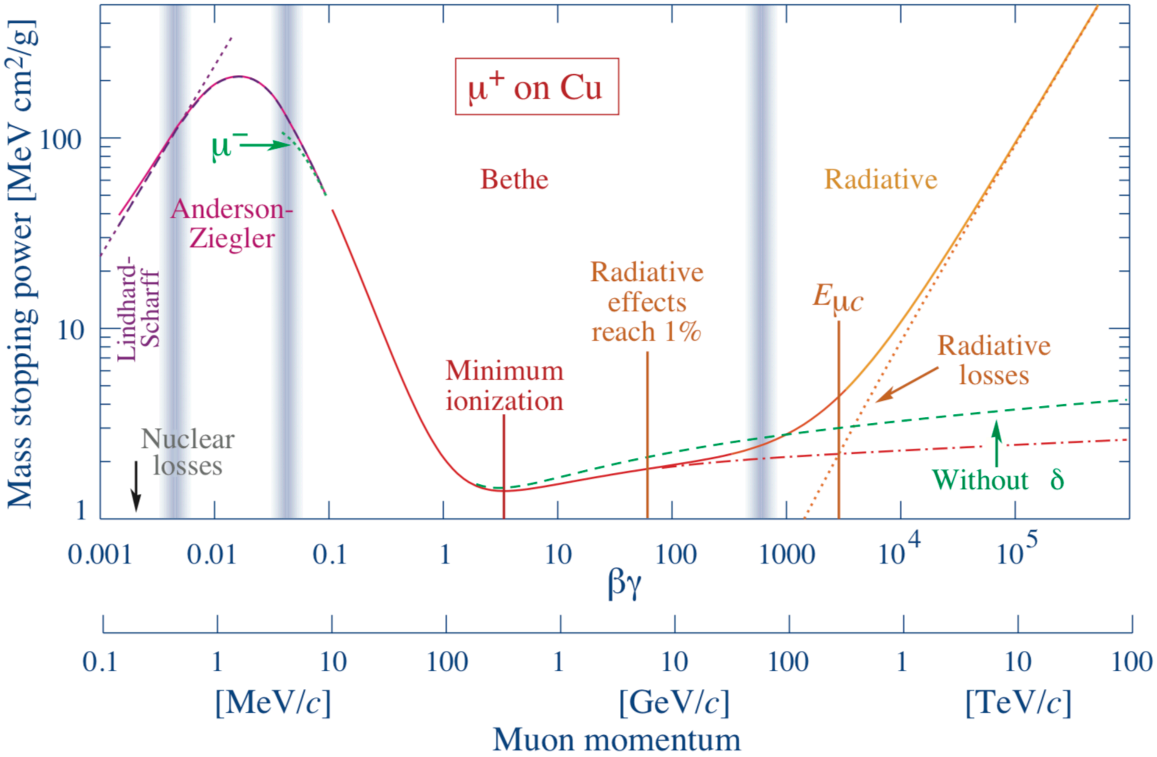
$$W_m = \frac{2m_e c^2 \beta^2}{1-\beta^2}$$

$$\frac{S}{\rho} = \sum_i w_i \left(\frac{S}{\rho}\right)_i$$

w_i -> fraction by weight of the ith element for mixtures

A 10 MeV proton loses about the same amount of energy in 1 g/cm^2 of copper as in 1 g/cm^2 of aluminum or iron

The Bethe-Bloch formula (Stopping Power)



M. Tanabashi et al. (Particle Data Group), (2018) Phys. Rev. D 98, 030001

Mass stopping power for positive muons in copper as a function of $\beta\gamma = p/Mc$.

μ^- illustrate the "Barkas effect" – dependence of the stopping power on projectile charge at very low energies

dE/dx in the radiative region is not simply a function of β

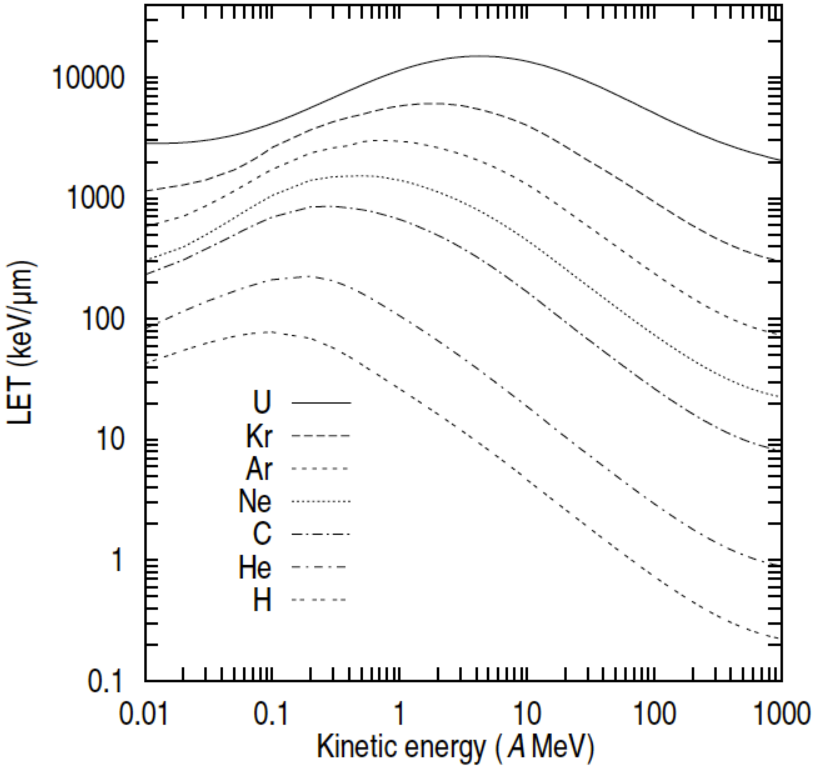
The Bethe-Bloch formula (Stopping Power)

The Barkas effect

Barkas Formula:

$$z_{eff} = z(1 - e^{-125\beta z^{2/3}})$$

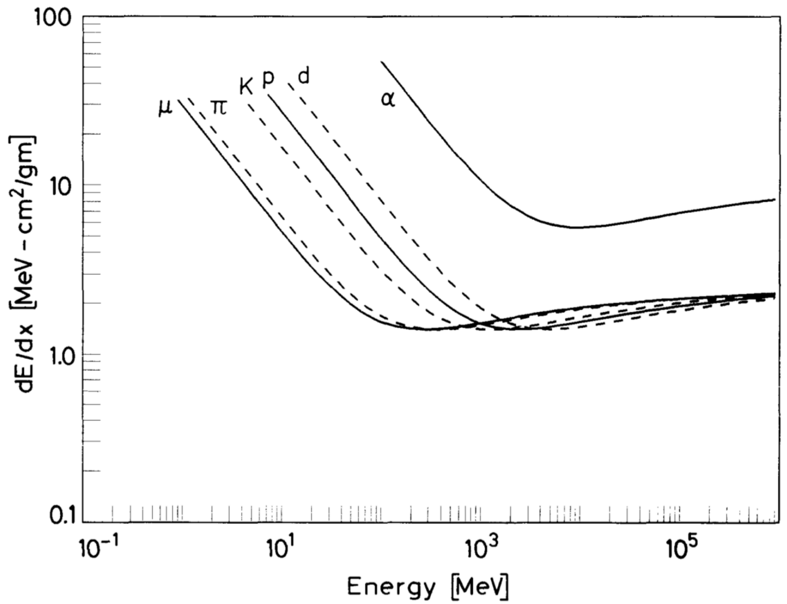
Heavier ions at low energies collect electrons from surrounding material thus rapidly decreasing its z_{eff}



Schwab (1991) PhD thesis, University of Giessen

The Bethe-Bloch formula (Stopping Power)

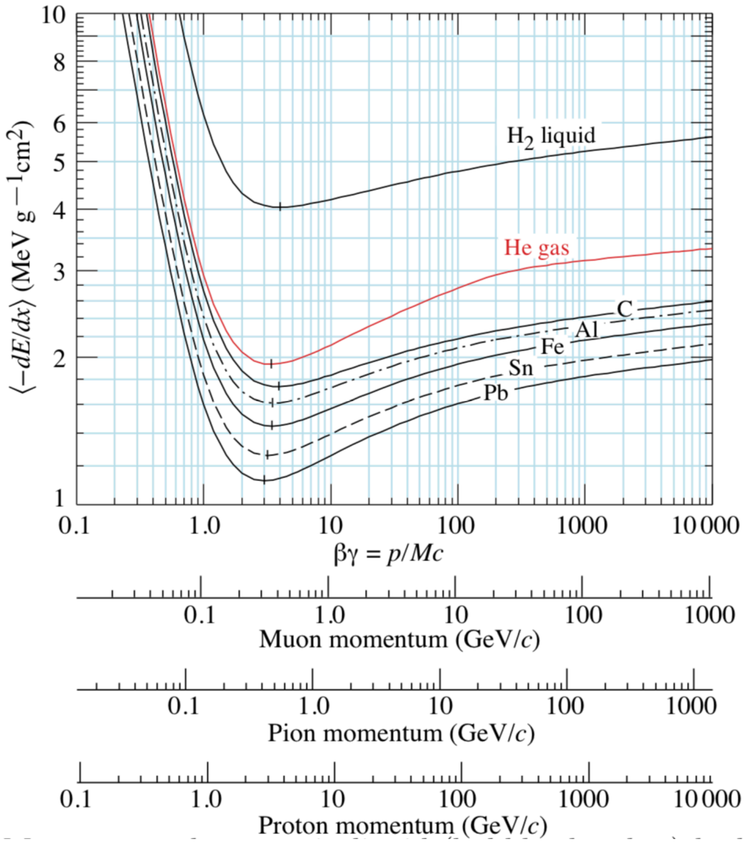
Energy dependence



W.R. Leo, (1994) Techniques for Nuclear and Particle Physics Experiments, Springer-Verlag

- At non-relativistic energies, dE/dx is dominated by the overall $1/\beta^2$ factor and decreases with increasing velocity until about $v = 0.96\ c$, where a minimum is reached (MIPs).
- The minimum value of dE/dx is almost the same for all particles of the same charge.
- As the energy increases beyond this point, the term $1/\beta^2$ becomes almost constant and dE/dx rises again due to the logarithmic dependence.

The Bethe-Bloch formula (Stopping Power)



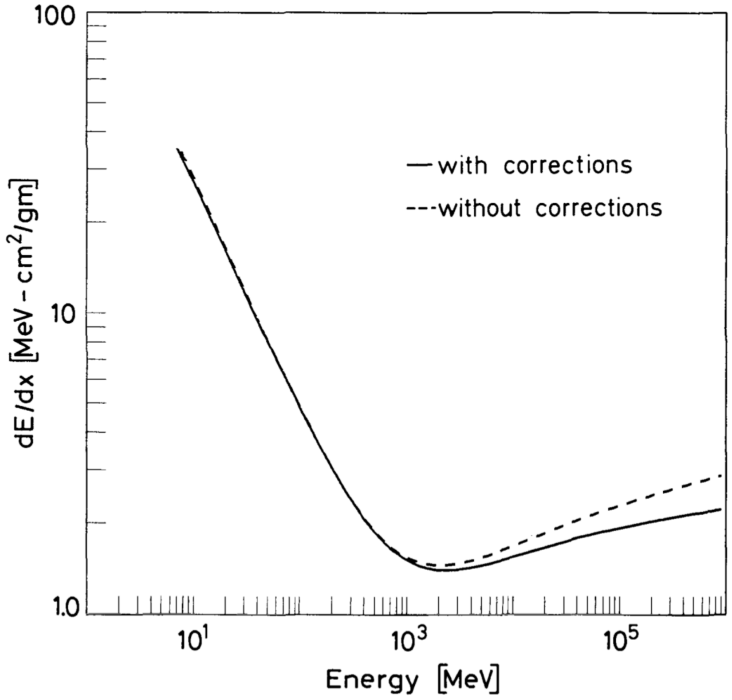
Mean energy loss rate in liquid hydrogen, gaseous helium, carbon, aluminum, iron, tin, and lead.

- Similar (slow decrease) rates of energy loss
- Density-effect correction for Helium – $\delta(\beta\gamma)$
- Broad minima drops from $\beta\gamma = 3.5$ to 3.0 as Z goes from 7 to 100
- Most relativistic particles (e.g., cosmic-ray muons) have mean energy loss rates close to the minimum (minimum ionizing particles - MIPs)

M. Tanabashi et al. (Particle Data Group), (2018) Phys. Rev. D 98, 030001

The Bethe-Bloch formula (Stopping Power)

The shell and density correction

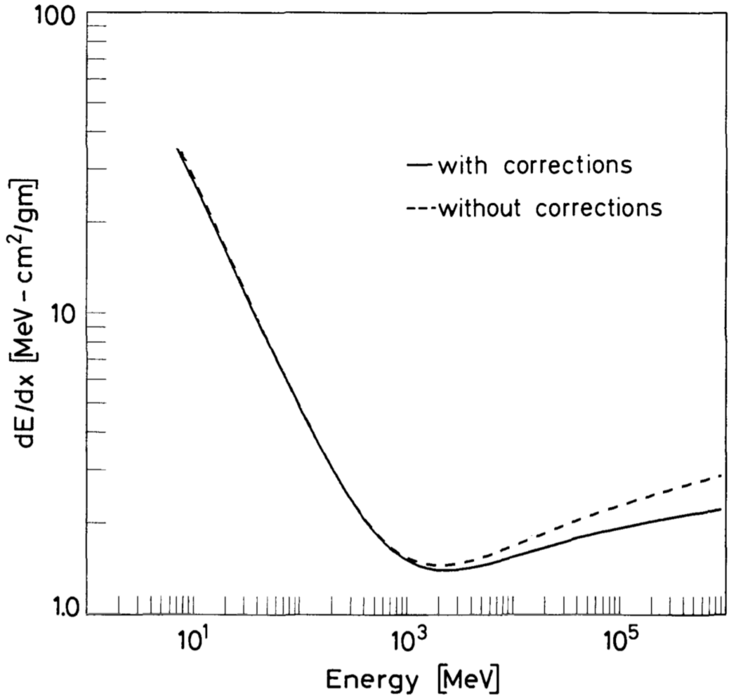


- The density effect arises from the fact that the electric field of the particle tends to polarize the atoms along its path
- Electrons far from the path are shielded form the full electric field intensity and the collisions contribute less to the energy loss
- Depends on the velocity of the particle and the density of the material (the higher both variables the stronger is the effect)

W.R. Leo, (1994) Techniques for Nuclear and Particle Physics Experiments, Springer-Verlag

The Bethe-Bloch formula (Stopping Power)

The shell and density correction



- The shell effect arises when the velocity of the incident particle is comparable or smaller than the orbital velocity of the bound electrons.
- At such energies, the assumption that the electron is stationary with respect to the incident particle is no longer valid and the Bethe-Bloch formula breaks down.
- The correction is very small!

W.R. Leo, (1994) Techniques for Nuclear and Particle Physics Experiments, Springer-Verlag

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- Fluence, stopping power and dose
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Fluence, stopping power and dose

Definitions

Fluence $$\Phi \equiv \frac{dN}{dA} \quad \frac{\text{protons}}{\text{cm}^2}$$

Fluence rate $$\dot{\Phi} \equiv \frac{d\Phi}{dt} \quad \frac{\text{protons}}{\text{cm}^2\ \text{s}}$$

Mass stopping power $$\frac{S}{\rho} \equiv -\frac{1}{\rho}\frac{dE}{dx} \quad \frac{\text{MeV}}{\text{g/cm}^2}$$

Dose $$D \equiv \frac{\text{J}}{\text{kg}}$$

(B. Gottschalk) H. Paganetti 2012

Fluence, stopping power and dose

The fundamental equation

$$D \equiv \frac{\text{energy}}{\text{mass}} = \frac{-(dE/dx) \times dx \times dN}{\rho \times dA \times dx}$$

$$D = 0.1602\ \Phi \frac{S}{\rho}\ \text{Gy}$$

Φ in Gp/cm^2

S/ρ in Mev/(g/cm^2)

(B. Gottschalk) H. Paganetti 2012

Fluence, stopping power and dose

Dose rate and beam current

$$\dot{D} = \frac{i_p}{A} \frac{S}{\rho} \quad \frac{\text{Gy}}{\text{s}}$$

i_p/A in **nA**/cm^2

S/ρ in MeV/(g/cm$^{2)}$

For a 170 MeV proton beam in water:

- current density of 0.0033 nA/cm^2
- S = 5 MeV/(g/cm^2)

$\Longrightarrow \dot{D} = 0.017$ Gy/min (typical radiotherapy rate)

(B. Gottschalk) H. Paganetti 2012

Fluence, stopping power and dose

Dose rate and beam current

- current density of 0.0033 nA/cm^2
- Targets with several cm^2 require proton currents entering the treatment head in the order of nA
- The tabulated stopping power is somewhat greater that the absorbed dose (a fraction goes into neutral secondaries – γ-rays and neutrons ranging further)
- The dose rate must never be used to estimate the dose delivered to the patient!! (dosimeters needed!)

(B. Gottschalk) H. Paganetti 2012

Accelerator and beam delivery (passive scattering)

Dose rate and beam current

$$\langle \dot{D} \rangle = \varepsilon f_{\mathrm{BP}} f_{\mathrm{MOD}} \frac{i_{\mathrm{p}}}{A} \left(\frac{S}{\rho} \right) \quad \frac{\mathrm{Gy}}{\mathrm{s}}$$

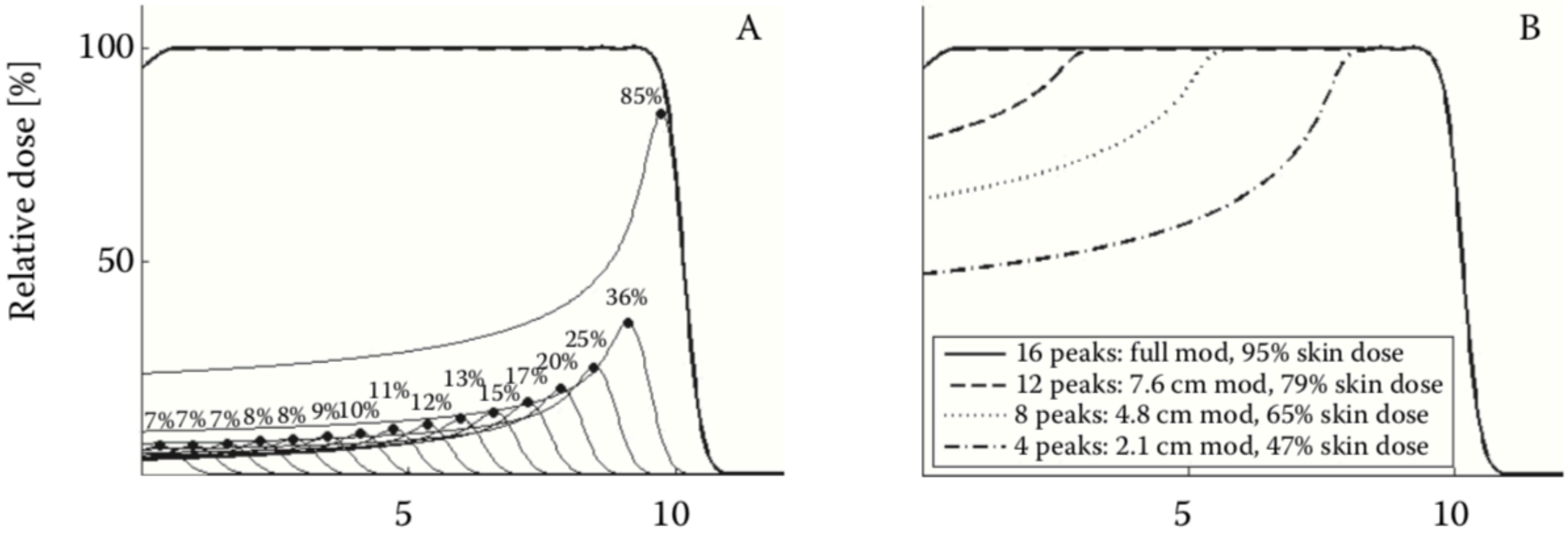
$\varepsilon =$ efficiency (single scattering ~ 0.05; double scattering ~ 0.45)

$f_{BP} =$ peak-to-entrance ratio of pristine $\mathrm{BP} \sim 3.5$ (energy independent)

$f_{MOD} =$ fractional dwell time of the deepest step (range modulation)

(B. Gottschalk) H. Paganetti 2012

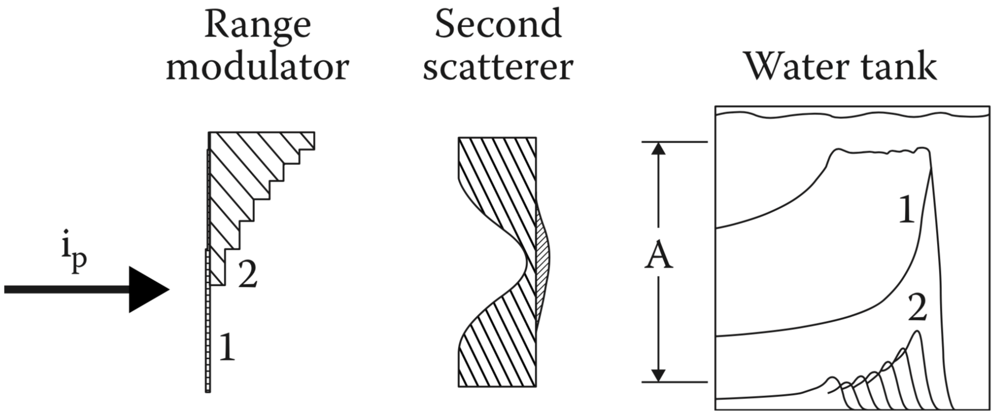
Accelerator and beam delivery (passive scattering)

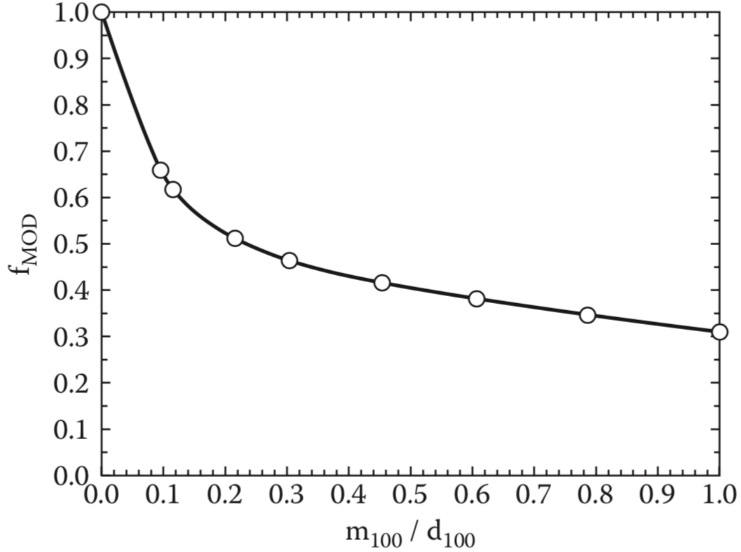


(R. Slopsema) H. Paganetti 2014

Fluence, stopping power and dose

Dose rate and beam current

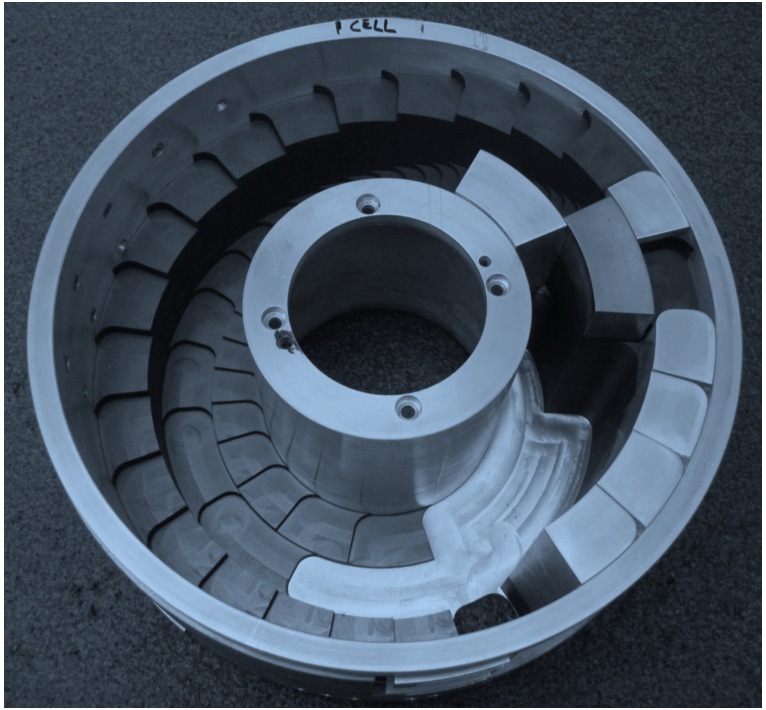




- For zero modulation, $f_{MOD} = 1$
- The dose rate is proportional to inverse area, but not to thickness of the modulator
- The deepest BP already delivers considerable dose to the entire volume (passive or active)

(B. Gottschalk) H. Paganetti 2014

Accelerator and beam delivery (passive scattering)



(R. Slopsema) H. Paganetti 2014

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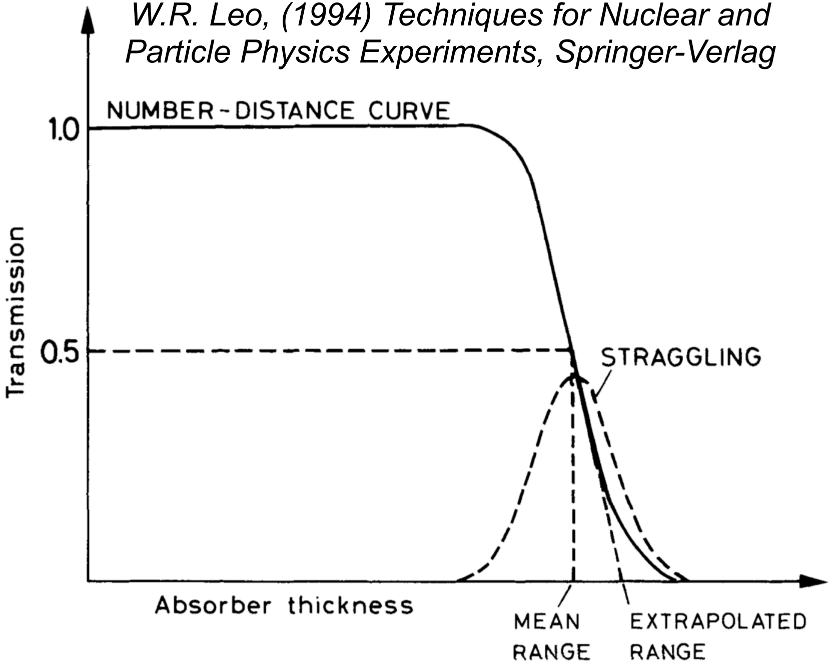
Range

- Range depends on the **type of material**, **particle type** and its **energy**
- Range can be determined by passing a beam of particles at the **desired energy** through **different thicknesses** of the material and measure the **ratio of transmitted to incident particles**

Range

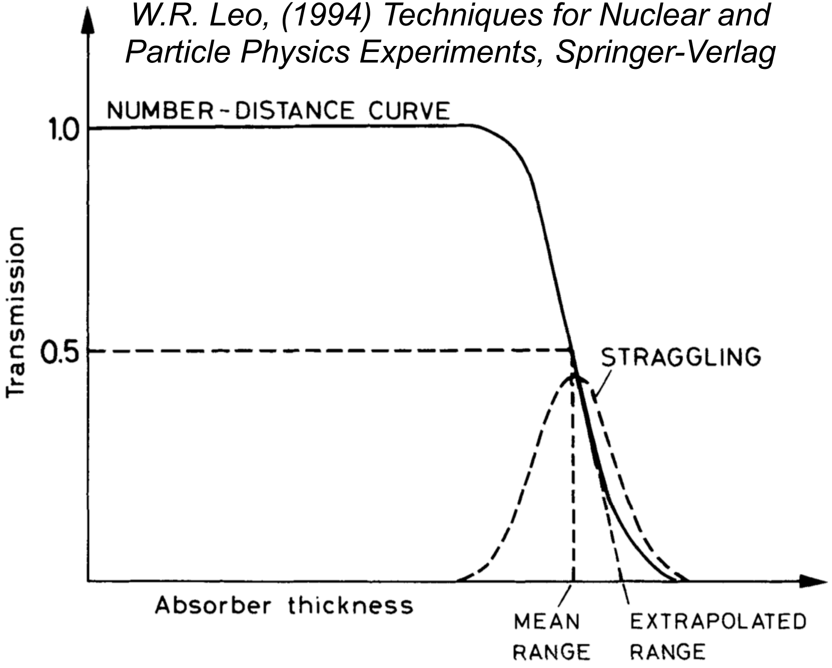


Typical range number-distance curve

- As the range is approached the ratio drops.
- The ratio does not drop immediately to the background level, but slopes down due to the non-continuous energy loss (statistical).
- Two identical particles with the same initial energy will *not* in general suffer the same number of collisions and energy loss.
- A measurement with an ensemble of identical particles shows a statistical distribution of ranges (range straggling), centered about a mean value (mean range).

Range

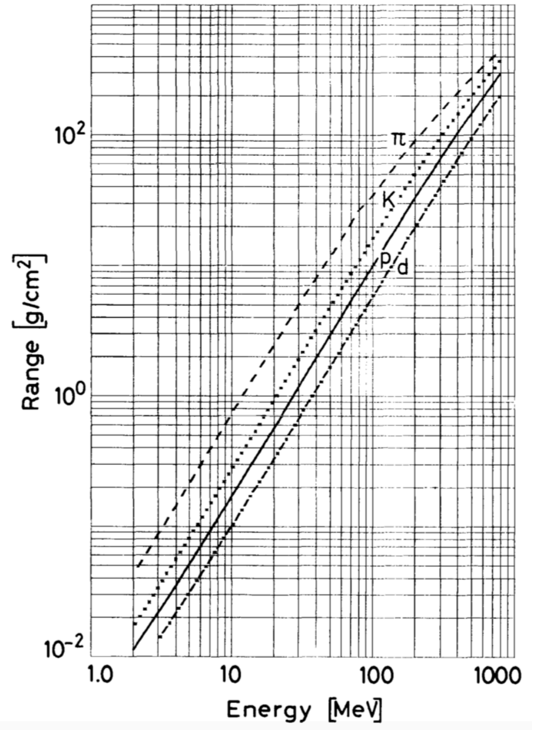
W.R. Leo, (1994) Techniques for Nuclear and Particle Physics Experiments, Springer-Verlag



Typical range number-distance curve

- **Mean range**: midpoint on the descending slope where half of the particles are absorbed
- **Extrapolated or pratical range**: the point at which the curve drops to the background level (tangent to the curve at the midpoint and extrapolating to the zero-level) – all particles are absorbed

Range



Calculated range curves of different heavy particles in aluminum

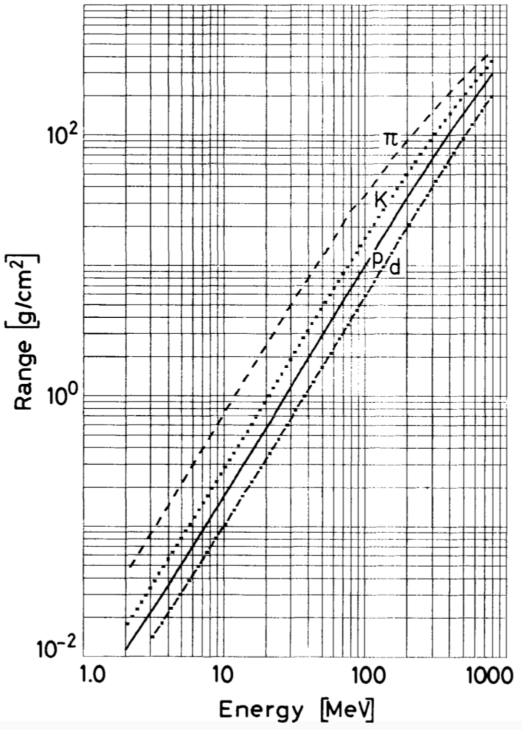
- Mean Range: $S(T_0) = \int_0^{T_0} \left(\frac{dE}{dx}\right)^{-1} dE$
- Approximate pathlength travelled and ignores MCS
- The strait-line range is smaller than the total zigzag
- MCS effect is small for heavy charged particles

- Semi-empirical formula:
- $R(T_0) = R_0(T_{min}) + \int_{T_{min}}^{T_0} \left(\frac{dE}{dx}\right)^{-1} dE$
- T_{min} = minimum energy at which dE/dx is valid
- $R(T_0)$ = empirically determined constant

W.R. Leo, (1994) Techniques for Nuclear and Particle Physics Experiments, Springer-Verlag

dkfz.

Range



- **Linear on the log-log scale**
- $R \propto E^b$
- The stopping power is dominated by the β^2 term,
- -dE/dx $-\mathrm{d}E/dx \propto \beta^{-2} \propto T^{-1}$
- Integrating: $R \propto T^2$
- A more accurate fit: $R \propto T^{1.75}$

Calculated range curves of different heavy particles in aluminum

W.R. Leo, (1994) Techniques for Nuclear and Particle Physics Experiments, Springer-Verlag

Range

Practical example: A beam of 600 MeV protons can be "lowered" in energy by passing it through a block of material such as copper and then "cleaned" using a series of analyzing magnets. What thickness of copper would be required to lower the average energy of this beam to 400 MeV?

$$\Delta x = -\int_{600}^{400} \left(\frac{dE}{dx}\right)^{-1} dE$$

Simple rectangular integration with energy intervals of 20 MeV and *dE/dx* evaluated in the middle of each interval

Range (MeV)	$\frac{1}{\rho}\frac{dE}{dx}$	$\Delta x = \Delta E\left(\frac{1}{\rho}\frac{dE}{dx}\right)^{-1}$
600 – 580	1.768	11.31
580 – 560	1.791	11.17
560 – 540	1.815	11.02
540 – 520	1.841	10.86
520 – 500	1.870	10.69
500 – 480	1.901	10.52
480 – 460	1.934	10.34
460 – 440	1.971	10.15
440 – 420	2.012	9.94
420 – 400	2.056	9.73

$\Delta x_{total} = 105.73\ \mathrm{g/cm^2} = 11.88\ \mathrm{cm}$

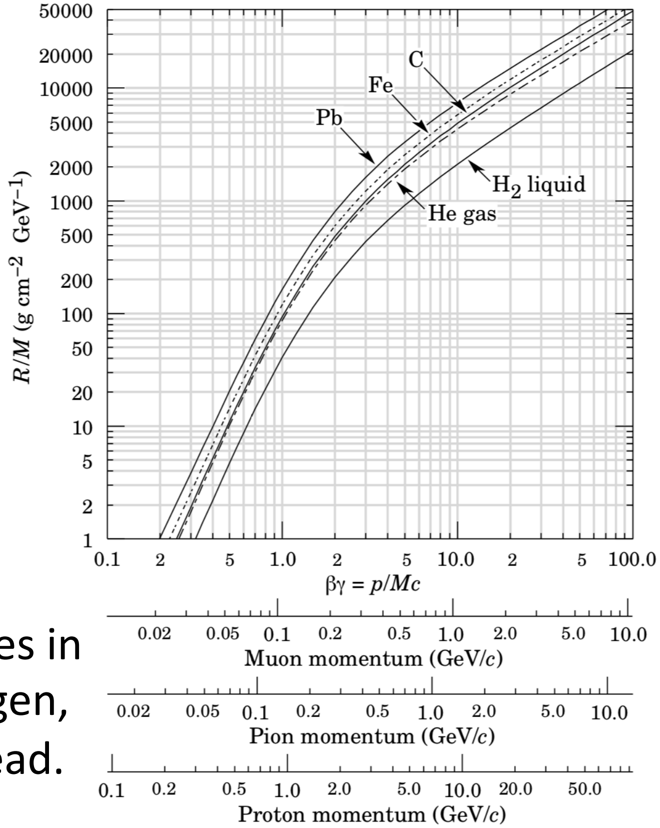
W.R. Leo, (1994) Techniques for Nuclear and Particle Physics Experiments, Springer-Verlag

Range

Bragg-Kleeman rule

For the same particle in different materials:

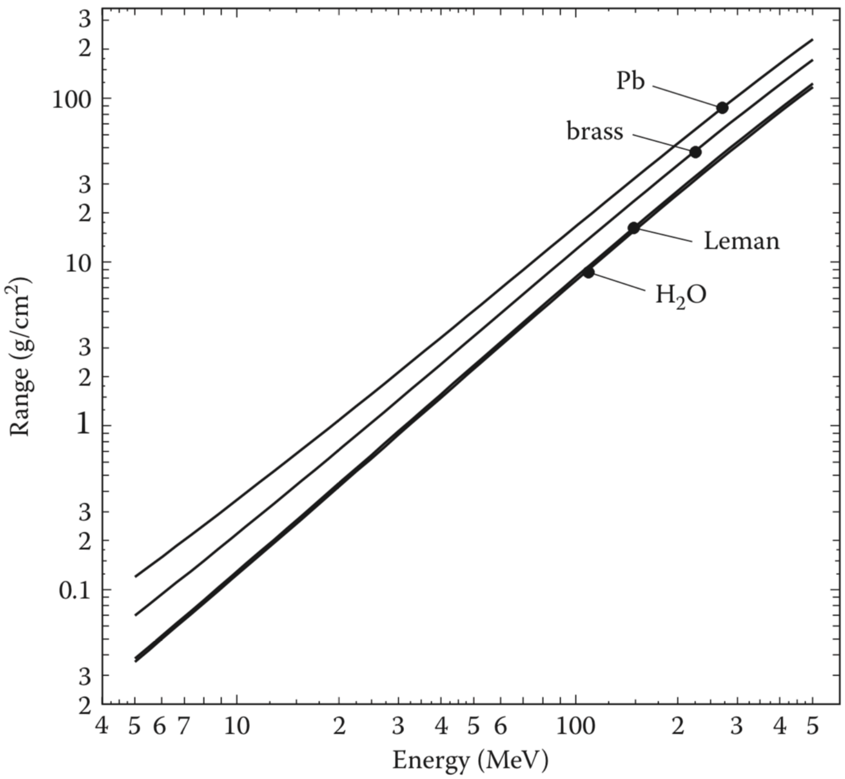
$$\frac{R_1}{R_2} = \frac{\rho_2}{\rho_1}\frac{\sqrt{A_1}}{\sqrt{A_2}}$$



Range of heavy charged particles in liquid (bubble chamber) hydrogen, helium gas, carbon, iron, and lead.

M. Tanabashi et al. (Particle Data Group), (2018) Phys. Rev. D 98, 030001

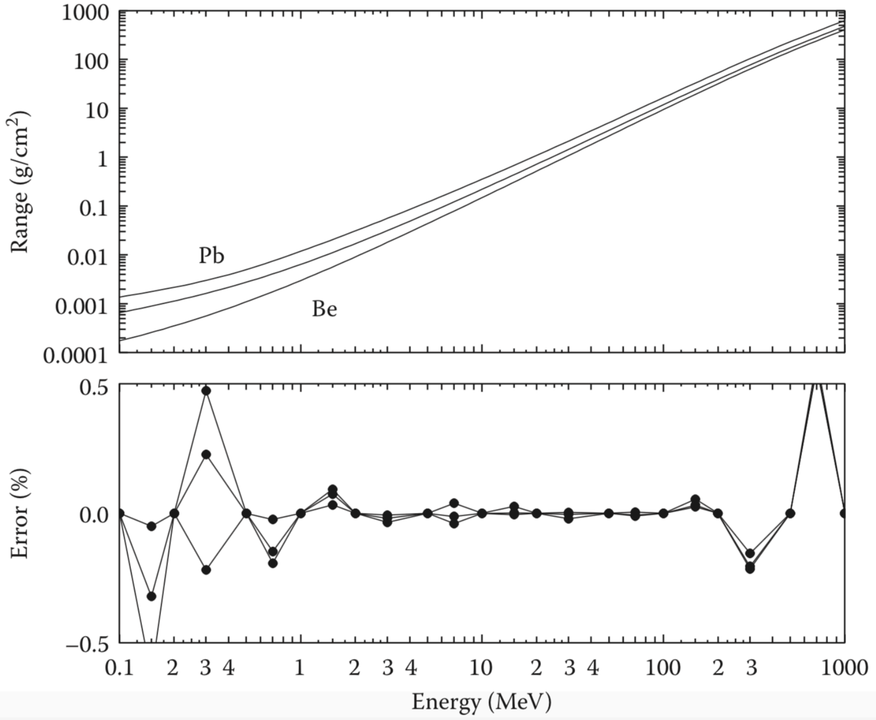
Range



- Proton range–energy relation around the clinical regime for four useful materials.
- At a given energy, the range expressed in g/cm^2 is greater (the stopping power is lower) for heavy materials.

(B. Gottschalk) H. Paganetti 2014

Range



- Range–energy relation in Be, Cu, and Pb and a test of cubic spline interpolation.
- Log-log graph for three materials over an extended region.
- Lines eventually curve. From 3-300 MeV (relevant energies) they are nearly straight, and for Be, Cu, and Pb, nearly parallel.
- Good candidates for *cubic spline interpolation.*
- Only thirteen input values for each material at 0.1, 0.2, 0.5, 1, ..., 500, 1000 MeV are required.

(B. Gottschalk) H. Paganetti 2012